Two Dimensional Projectile Motion

List the possible forces that affect a moving object:
1. velocity magnitude
2. velocity direction
3. path of the object in motion
4. wind resistance
5. gravity

Components of vectors can be used to predict the motion of objects thrown into the air. Finding the displacement, velocity, and acceleration of an object in flight is accomplished by breaking the motion of the object into its vector quantities.
1. Resolve the vectors into their simpler components.
2. Apply simpler one-dimensional equations for each component.
3. Recombine the components to determine the resultant.

projectiles – object thrown or launched into the air and are subject to gravity

projectile motion – free fall motion (two dimensional) with an initial horizontal velocity (note: find a better definition)

A projectile in motion will have vertical motion and horizontal motion. Taking only these two factors into consideration, the path of the projectile will be **parabolic**. A third factor considered is air resistance that will slow the projectile and this factor will cause the true path to be non parabolic.

Note: For the majority of our projectile motion problems, we will not consider air resistance and we will consider that the horizontal velocity is constant.

What is the single most important factor that affects the vertical velocity? Gravity.

parabola – a curve in which every point is the same distance from a fixed point call the **focus** as it is from a fixed line called the **diretrix** (Every hear the term parabolic reflector?)

What are some factors that affect air resistance?
1. density
2. air speed
3. humidity
4. elevation
5. barometric pressure

Two parts for solving projectile motion:

1. **Vertical Motion** – falling from rest:

$$v_{y,f} = -g\Delta t \qquad v_{y,f}^2 = -2g\Delta y \qquad \Delta y = -\tfrac{1}{2}g(\Delta t)^2$$

If an object is falling straight down, we can disregard air resistance and g = 9.81 m/s^2. Assuming that the object is starting at rest, then $\boldsymbol{v_{y,i}}$ **= 0 m/s**. ***v* = velocity**, subscript $_y$ = magnitude or position on ***y*-axis**, and subscript $_i$ = initial or beginning (velocity). Since **g** is negative in these equations, then the displacement on the ***y*-axis** will also be negative (down).

2. **Horizontal Motion** - always constant

$$v_x = v_{x,i} = \text{constant}$$

To find the velocity of a projectile at any point, find the vector sum of the components of the velocity at that point. Two parts:

1. Use Pythagorean's theorem to find the magnitude of the velocity.
2. Use the trig functions to find the direction of the velocity.

Analyze the motion of objects launched at an angle.

The object will have a vertical motion and a horizontal motion and the initial velocity vector will have an angle θ to the horizontal.

To analyze the motion of the object:

1. The object's motion must be resolved into it components.
2. Sine and cosine functions are used to resolve the horizontal and vertical components of the initial velocity.

   $\boldsymbol{v_{x,i} = v_i(\cos\theta)}$ → horizontal or "**x**" component; always constant

   $\boldsymbol{v_{y,i} = v_i(\sin\theta)}$ → vertical or "**y**" component; accelerates or decelerates due to gravity.

3. Substitute the values into the kinematics' equations from chapter 2 on page 58 to obtain equations that are used to analyze two-dimensional motion, the motion of an object launched at an angle.

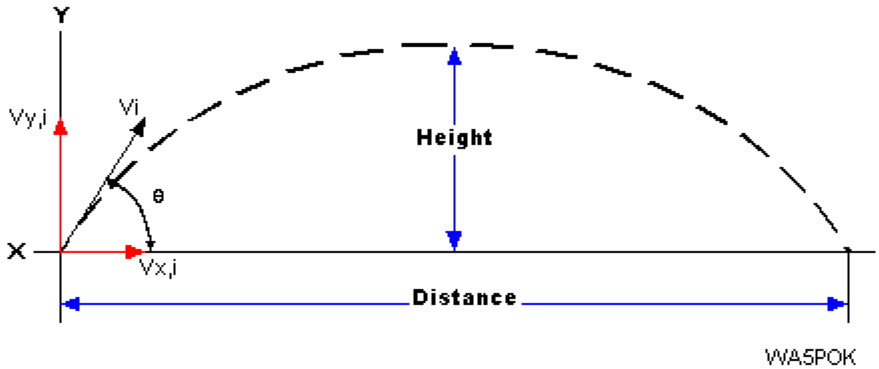
Kinematics Equation for one dimension motion	Kinematics Equations for two dimension motion
$\Delta x = \Delta v ? \Delta t$	$\Delta x = v_i(\cos\theta)\Delta t$
$v_f = a ? \Delta t$	$v_f = -g ? \Delta t$
$\boldsymbol{v_f = v_i + a(\Delta t)}$	$\boldsymbol{v_{y,f} = v_i(\sin\theta) - g\Delta t}$
$\boldsymbol{v_f^2 = v_i^2 + 2a\Delta y}$	$v_{y,f}^2 = v_i^2(\sin)^2 - 2g\Delta y$ *
d = ½ gt^2	$\boldsymbol{\Delta y = -\tfrac{1}{2}g(\Delta t)^2}$
d = ½ gt^2	$\boldsymbol{\Delta y = v_i(\sin\theta)\Delta t - \tfrac{1}{2}g(\Delta t)^2}$ *

Notes: 1. The change in sign in front of **g** indicates the object is moving down towards Earth.
2. The vertical motion upward is similar to that of an object that is thrown straight up with an initial velocity.

*** Use these equations if the object has an initial starting velocity. Example: the object is thrown downward or upward.**

A projectile is launched with a velocity of **75 m/s** at an angle **?** of **60°**. Draw the path of the projectile and resolve the vector into the **x** and **y** components at the moment of launch. Calculate the maximum height, time to maximum height and distance the projectile travels.

a) The Path:



b) Resolve the vector into it's horizontal and vertical components:

		Horizontal			Vertical
$V_{x,i}$	=	V_i(cos?)	$V_{y,i}$	=	V_i(sin?)
$V_{x,i}$	=	75 m/s (cos60°)	$V_{y,i}$	=	75 m/s (sin60°)
$V_{x,i}$	=	**37.5 m/s**	$V_{y,I}$	=	**65.0 m/s**

Lets gather our known information about this problem.

1. V_i is **75 m/s**.
2. $V_{x,i}$ is **37.5 m/s**, the **X** component (constant horizontal velocity) of the projectile.
3. $V_{y,i}$ is **65.0 m/s**, the **Y** component (vertical velocity at launch) of the projectile.
4. Gravity (**g**) is **9.81 m/s**.
5. The vertical velocity at the maximum height (apex) is **0 m/s**.
6. The time to the apex is **one way time**.

c) Maximum Height:

The equation to calculate the maximum height of the projectile is $Dy = v_i(\sin q)Dt - \frac{1}{2}g(Dt)^2$. The missing item to solve for the height is the time. Use the equation $v_f = g?Dt$ to find the time. Since the velocity at the apex is 0 m/s the final vertical ($V_{x,i}$) velocity as it contacts the surface is **65.0 m/s**. What goes up will come down at the same magnitude.

$$Dt = \frac{v_f}{g} \qquad Dt = \frac{65m/s}{9.81m/s^2} \qquad Dt = 6.6\ s$$

Now solve for the maximum height.

$$Dy = v_i(\sin q)Dt - \frac{1}{2}g(Dt)^2 \qquad Dy = (65\ m/s)(6.6s) - ½\ (9.81\ m/s2)(6.6\ s)^2 \qquad Dy = 213.3\ m$$

d) Time to maximum height:

This was done in step "**c**" as it was needed to solve for the maximum height. The answer from above is **6.6 s**. This is the one-way time to the apex.

e) Horizontal distance:

To find the horizontal distance, use the equation $Dx = Dv?Dt$. It takes the projectile the same amount of time to rise as it does to fall. The one-way time is **6.6 s** and therefore the two-way time and Dt is **13.2 s** that is also the time the projectile travels its curved path. Dv will be the "**x**" or horizontal component used in the equation and the value as resolved is **37.5 m/s**.

$$Dx = Dv?Dt \qquad Dx = (37.5\ m/s)(13.2s) \qquad Dx = 495\ m$$

Pick the step(s) is this example to assist you in solving two-dimensional problems you encounter.

Frames of Reference

Velocity measurements require a **frame of reference** in the background. The **movement** of an object relative to a frame of reference can be measured by an observer. The observer must refer to something that is assumed to be fixed in place. For travel on the surface of the Earth, the most common frame of reference is a point on **Earth**.

Different Frames of reference for the same object may produce different results. Different velocities may result and different displacements may be calculated. Think the $125 million dollar mistake.

Relative Velocity - The movement of a particle relative to a coordinate system that is also moving relative to a second coordinate system that is also moving.

$$V_{pB} = V_{pA} + V_{AB}$$

You are swimming 1.2 m/s in a river parallel to the flow of the river. You have a velocity relative to the river. The river has a velocity of 4.8 m/s relative to the shore. Establish an equation to solve your velocity relative to the shore.

First: define the terms:

1. your velocity relative to the water = Vyw;
2. water velocity relative to the shore = Vws;
3. your velocity relative to the shore = Vys

Second: derive the formula:

Vys = Vyw + Vws

Third: plug in the numbers and solve.

6 m/s = 1.2 m/s + 4.8 m/s

Forth: Check your formula.

Check your math.

Einstein's Theory of Relativity

What is the constant that is added to the above formula to enhance the accuracy? **The speed of light in a vacuum that is 3 x 10^8 m/s.**

The above formula is all we need when dealing with large objects that are "relatively" slow. Mathematical calculations will not show any significant changes. When working with the speed of electrons in a nuclear reaction or working with the movement of celestial objects (stars, planets, and galaxies) then the above formula is modified to include "**c**."

$$\text{Vys} = \frac{V_{YW} + V_{WS}}{1 + V_{YW}V_{WS}/C^2}$$

The above equation and this equation are the same because $\mathbf{V_{YW}}$ and $\mathbf{V_{WS}}$ are so much smaller than **c**. For very high speeds such as the speed of an election, etc. then the difference between the two equations in very significant. What is "**c**"? Use the above formula to recalculate your relative velocity to the shore. What is the difference?