Momentum and Collisions

Momentum and Impulse

Momentum is used to describe the motion of an object before and after a force acts on it. Momentum is also used to explain what happens to object during a collision.

Momentum – a **vector quantity** (because velocity has both speed and direction) that is a **product** between the **mass** and **velocity** of an object. Momentum is calculated by the following equation.

$$\mathbf{p} = m\mathbf{v}$$

p = momentum, units are kg • m/s
m = mass in kilograms (kg)
v = velocity in m/s

The little "**p**" is from the term **progress** defined as "**the quantity of motion with which a body proceeds in a certain direction**."

The momentum of an object is directly proportional to the objects mass and velocity. The **faster** the object moves, the **greater the momentum**. As object rolling down hill, it is picking up speed because the force of gravity will accelerate the object at a constant rate. The object is "gathering momentum." The greater the momentum of the snow ball rolling down the hill, then the more force is required to bring the snow ball to a stop.

To change momentum takes a change in force and time, therefore momentum is closely related to force.

What is the effect on your hand when you catch a fast moving ball?
What is the effect on your hand when you catch a slow moving ball?

Back to Newton's second law: **F =** ***ma*** is also written in the following form: $\mathbf{F} = \frac{\Delta p}{\Delta t}$ as the net external force and the time interval required to make the change in momentum.

Impulse – Momentum Theorem

$$\mathbf{F}\Delta t = \Delta\mathbf{p} \quad \text{or} \quad \mathbf{F}\Delta t = \Delta\mathbf{p} = m\mathbf{v_f} - m\mathbf{v_i}$$

Force **X** time interval = change in momentum

This theorem states that an external force applied over a certain time interval will change an object's momentum. A small force applied over a long time will have the same effect in momentum as a large force applied over a short time assuming constant forces. $\mathbf{F}\Delta t = \Delta\mathbf{p}$ is the **impulse – momentum theorem** because on the left side of the equation, $\mathbf{F}\Delta t$ is the impulse of the force for the time interval Δt.

Impulse – for a constant external force, **impulse** is the product between the force and the time over which the force acts on an object

This equation demonstrates why it is so important to "follow through" in sports. When contact is made with the ball by the bat in baseball, the bat is in contact for a longer time with the ball adding causing a greater change in the momentum of the ball.

Momentum is useful to calculate the stopping distances for moving vehicles to determine safe following distances.

Conservation of Momentum

What happens when there is a collision between objects? Momentum is conserved. The momentum just doesn't go away, it is transferred between objects that collide.

When two objects collide, the total momentum of both objects before the collision is the same after the collision as expressed in the following equation. **A**, **B** are the two objects that collide and **i** and **f** are the initial and final momentums of the respective objects.

$$\mathbf{P}_{A,i} + \mathbf{P}_{B,i} = \mathbf{P}_{A,f} + \mathbf{P}_{B,f}$$

Therefore the **law of conservation of momentum** is stated: **"as the total momentum of all objects interacting with one another remains constant regardless of the nature of the forces between the objects."** This law holds true if the objects collide or the objects push away from each other as in a person jumping or two ice skaters pushing each other apart. The law of conservation of momentum is also express with the following equation.

$$m_1\mathbf{v}_{1,i} + m_2\mathbf{v}_{1,i} = m_1\mathbf{v}_{1,f} + m_2\mathbf{v}_{2,f}$$

Total initial momentum = total final momentum

Newton's third law of motion – for every action there is an equal and opposite reaction and **leads to the conservation of momentum**.

Think of holding and firing a gun. Before the gun is fired, the velocity of the gun is zero and the velocity of the bullet is zero. After the gun is fired, the small mass of the bullet is propelled forward with a high velocity and the large mass of the gun is propelled backward with a low velocity. The momentum is the same both before and after firing the gun.

Elastic and Inelastic Collisions

Objects collide and bounce off of one another. Objects collide and stick together.

Perfectly inelastic collision – a collision in which two objects stick together and move with a common velocity after colliding

An example of a perfectly inelastic collision is when an arrow strikes a target and sticks in it. The final mass is equal to the mass of the arrow and the target.

Two cars of m_1 and m_2 collide and they stick together after the collision. The stuck cars will move together with some common velocity, $\mathbf{v_f}$**,** after the collision. Remember $\mathbf{p} = m\mathbf{v}$. This **perfectly inelastic equation** is expressed with the following equation.

$$m_1\mathbf{v}_{1,i} + m_2\mathbf{v}_{2,i} = (m_1 + m_2)\mathbf{v}_f$$

Note: Pay attention to signs that indicate direction.

Kinetic energy is not constant in inelastic collisions when objects collide and stick together. **KE** is converted into internal energy (motion of the atoms) when an object deforms and some energy is converted into sound.

Elastic collision – a collision in which the total momentum and the total kinetic energy remains constant

In an elastic collision, the two objects will return to their original shape after the collision. Both the total momentum and total **KE** remain constant.

Most collisions are not perfectly inelastic because the objects do not usually stick together and continue to move as one object.

Most collisions are not elastic because when contact is made, one or more of the objects deforms. During Deformation **KE** may be converted into heat, sound, and/or internal **PE** and thus a decrease in **KE**.

Kinetic energy is **conserved** in elastic collisions because the total momentum and the total kinetic energy remain constant throughout the collision. This is illustrated in the equations:

$$m_1\mathbf{v}_{1,i} + m_2\mathbf{v}_{1,i} = m_1\mathbf{v}_{1,f} + m_2\mathbf{v}_{2,f}$$

$$\tfrac{1}{2}m_1\mathbf{v}_{1,i}^2 + \tfrac{1}{2}m_2\mathbf{v}_{1,i}^2 = \tfrac{1}{2}m_1\mathbf{v}_{1,f}^2 + \tfrac{1}{2}m_2\mathbf{v}_{2,f}^2$$